Technical Note: An Eaton and Kortum (2002) Model of Urbanization and Structural Transformation*

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1 Introduction

This technical note extends the model of Michaels et al. (2011) to introduce Eaton and Kortum (2002) heterogeneity within each sector, which facilitates a tractable analysis of bilateral transport costs. We show how the resulting model can be calibrated using data on employment by sector and location to determine productivity in each sector and location. We undertake this calibration for each year separately without imposing prior structure on the pattern of productivity growth over time. We show that the resulting calibrated productivities support the explanation for the stylized facts in Michaels et al. (2011) based on a combination of aggregate reallocation from agriculture to non-agriculture and differences in mean reversion in productivity between the two sectors.

2 Model

We consider an economy that features many locations indexed by $i = 1, \ldots, I$, labor and land as factors of production, and agricultural and non-agricultural sectors denoted by $K \in$

*This technical note provides additional supplementary material for Michaels, Redding and Rauch (2011) "Urbanization and Structural Transformation," London School of Economics, mimeograph.

$\{A, N\}$. While locations can differ from one another in terms of both their productivities and bilateral transport costs, the model remains tractable because of the stochastic formulation of productivity, which follows Eaton and Kortum (2002).

The economy as a whole is endowed with a measure L_t of workers, who are mobile across locations and are each endowed with one unit of labor that is supplied inelastically with zero disutility.[1] Employment varies across sectors and locations because of exogenous productivity differences. Population mobility ensures that workers in all populated locations receive the same real income in equilibrium.

Each location is endowed with a fixed measure H_i of land, which can be used residentially or commercially in agriculture and non-agriculture. Production in each location can be either completely specialized in one sector or incompletely specialized. Land in each location is owned by immobile landlords, who earn rents and consume with the same preferences as workers.

There are two key differences between the sectors in the model. First, we make the natural assumption that agriculture is more land intensive than non-agriculture. Second, we allow the two sectors to differ in terms of their distribution of productivities across locations. Together, these two features of the model allow the share of agriculture in employment to vary with population density. While our focus on agriculture and non-agriculture is motivated by our empirical findings of the importance of reallocation away from agriculture, the model could be extended to disaggregate non-agriculture into manufacturing and services.

The model captures the two main explanations for an aggregate reallocation of employment away from agriculture in the macroeconomics literature: (a) more rapid productivity growth in agriculture than in non-agriculture combined with inelastic demand across sectors and/or (b) non-homothetic preferences combined with a rise in real income that leads to a change in the relative weight of sectors in consumer preferences. Either of these explanations can induce an aggregate reallocation of employment away from agriculture, and our objective is not to distinguish between them, but rather to examine the implications of such an aggregate reallocation for the distribution of population across locations.

[1]Since labor force participation is not strongly related to population density in our data, the model abstracts from workers' labor force participation decision and assumes that total employment is equal to total population. While the analysis could be extended to allow each worker to have a fixed number of dependents, who consume but do not work, this would complicate the analysis without adding much insight.

2.1 Consumer Preferences

Preferences are defined over goods consumption (C_{jt}) and residential land use (H_{Ujt}) and are assumed for simplicity to take the Cobb-Douglas form:[2]

$$U_{jt} = \eta_{jt} C_{jt}^{\alpha} H_{Ujt}^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{1}$$

where η_{jt} captures residential consumption amenities in location j at time t.[3]

The goods consumption index is defined over consumption of agriculture (C_{Ajt}) and non-agriculture (C_{Njt}):

$$C_{jt} = \left[\psi_{At} C_{Ajt}^{\rho} + \psi_{Nt} C_{Njt}^{\rho}\right]^{\frac{1}{\rho}}, \qquad 0 < \kappa = \frac{1}{1-\rho} < 1, \quad \psi_{At}, \psi_{Nt} > 0, \tag{2}$$

where agriculture and non-agriculture are assumed to be complements ($0 < \kappa < 1$), which implies inelastic demand between the two goods.[4]

The demand parameters (ψ_{At}, ψ_{Nt}) capture the relative strength of consumer tastes for each good and may be a function of the common value of real income (v_t) across all locations ($\psi_{At} = \psi_A(v_t)$, $\psi_{Nt} = \psi_N(v_t)$), as in the non-homothetic specification of CES utility considered by Sato (1977).[5] Under our assumption of CES preferences, changes in relative productivity across sectors are isomorphic to changes in the relative weight of sectors in consumer preferences, in the sense that they have the same effects on equilibrium expenditure and employment shares. Since real income is the same across locations in an equilibrium characterized by population mobility, the demand weights $\psi_{At} = \psi_A(v_t)$ and $\psi_{Nt} = \psi_N(v_t)$ are common across locations, which enables us to determine an idiosyncratic component of productivity that varies across locations within each sector. However, the common demand weights $\psi_{At} = \psi_A(v_t)$ and $\psi_{Nt} = \psi_N(v_t)$ cannot be separately identified from aggregate differences in productivity across sectors using the employment data available to us. Therefore we normalize the demand weights to one ($\psi_{At} = \psi_A = 1$ and $\psi_{Nt} = \psi_N = 1$ for all t), so that the calibrated aggregate component of productivity in each sector captures both the common demand weights and aggregate productivity.

[2] For empirical evidence using U.S. data in support of the constant housing expenditure share implied by the Cobb-Douglas functional form, see Davis and Ortalo-Magne (2008).

[3] To simplify the exposition, throughout the following, we denote producing locations by i and consuming locations by j.

[4] In assuming inelastic demand between agriculture and non-agriculture, we follow a large literature in macroeconomics, as discussed in Ngai and Pissarides (2007).

[5] The relative weight of the two sectors in consumer preferences (ψ_{At}, ψ_{Nt}) may also capture differences in product quality across sectors and over time.

The goods consumption index for each sector K in location j at time t is defined over consumption $(c_{Kjt}(h))$ of a fixed continuum of products $h \in [0,1]$ (e.g. rice, wheat, corn etc for agriculture):[6]

$$C_{Kjt} = \left[\int_0^1 c_{Kjt}(h)^{\nu} dh \right]^{\frac{1}{\nu}}, \qquad \sigma = \frac{1}{1-\nu} > 1, \qquad K \in \{A, N\}, \tag{3}$$

where we make the plausible assumption that products within a given sector are substitutes for one another ($\sigma > 1$). While for simplicity we assume the same elasticity of substitution in both sectors, it is straightforward to relax this assumption.

2.2 Production

Production in each sector is modeled following Eaton and Kortum (2002), which we augment to introduce land as an additional factor of production and labor mobility across locations.[7] Each location i draws an idiosyncratic productivity $(z_K(h))$ for each product h within each sector K. Productivity is independently drawn across products, sectors and time from a Fréchet distribution:

$$F_{Kit}(z_K) = e^{-T_{Kit} z_K^{-\theta_K}}, \qquad K \in \{A, N\}, \tag{4}$$

where the scale parameter T_{Kit} determines average productivity for each sector-location-year; the shape parameter θ_K controls the dispersion of productivity across products within each sector-location-year. Variation in the productivity parameter (T_{Kit}) across sectors and locations determines the distribution of employment across locations within each sector and the distribution of population across locations.

Products within each sector are produced under conditions of perfect competition using labor and land according to a Cobb-Douglas production technology.[8] Output can be traded across locations subject to iceberg variable trade costs, where $d_{Kji} > 1$ units of a product

[6] We use "sector" or "good" to refer to the broad categories of agriculture and non-agriculture and reserve "product" for the various types of good within each sector (e.g. rice and wheat within agriculture).

[7] While Yi and Zhang (2010) distinguish between agriculture and non-agriculture in a model of the aggregate economy following Eaton and Kortum (2002), we introduce population mobility and endogenous land use across many disaggregate regions within an economy. While Holmes, Hsu and Lee (2011) develop a two-region version of Bernard, Eaton, Jensen and Kortum (2003) to analyze endogenous innovation and location decisions, we distinguish between agriculture and non-agriculture and examine the implications of structural transformation for urbanization.

[8] While we assume a Cobb-Douglas production technology and follow a long line of research in macroeconomics in focusing on changes in aggregate productivity or relative demand across sectors as the sources of reallocation, another potential explanation for this reallocation involves distinguishing between land and labor-augmenting technological change using a more flexible production technology.

in sector K must be shipped from location i in order for one unit to arrive in location j. Therefore the cost to a consumer in location j of purchasing one unit of a product h within sector K from location i is as follows:

$$p_{Kjit}(h) = \frac{w_{it}^{\mu_K} r_{it}^{1-\mu_K} d_{Kji}}{z_{Kit}(h)}, \qquad 0 < \mu_K < 1, \;\; K \in \{A, N\}, \tag{5}$$

$$d_{Kji} = \tau_{ji}^{\delta_K}, \qquad \delta_K > 0,$$

where w_{it} and r_{it} denote the wage and land rent, respectively, in location i at time t.

We assume that agriculture is land intensive relative to non-agriculture ($\mu_A < \mu_N$), which together with sectoral differences in the pattern of calibrated productivities (T_{Kit}) across locations rationalizes sectoral differences in the pattern of employment across locations as an equilibrium of the model. We follow a long gravity equation literature in assuming that bilateral trade costs are a log linear function of bilateral distance (τ_{ji}), where the distance coefficient δ_K parameterizes the elasticity of trade costs with respect to distance. In our calibration of the model, we examine the extent to which observed changes in employment can be explained simply by differences in aggregate growth rates and mean reversion of productivity across sectors. Therefore, consistent with empirical findings in the gravity equation literature, we assume that the distance coefficient is constant over time.[9]

2.3 Expenditure Shares

Given the above specification of preferences and technology, consumer equilibrium can be characterized by first determining shares of expenditure across locations within each sector, and next determining shares of expenditure across sectors.

Price distributions: Before determining expenditure shares, we provide a characterization of the distribution of prices, which follows Eaton and Kortum (2002). Note that a given product is homogeneous in the sense that one unit of that product is the same as any other unit of that product. Therefore the representative consumer in a given location sources each product from the lowest-cost supplier to that location:

$$p_{Kjt}(h) = \min\{p_{Kjit}(h); i = 1, \ldots, I\}, \qquad K \in \{A, N\}. \tag{6}$$

[9] While the level of trade costs has fallen over time, the empirical gravity equation literature finds that the elasticity of trade costs with respect to distance has remained remarkably constant over time, as surveyed in Disdier and Head (2008). Since we calibrate the model for each year separately, it would be straightforward to relax the assumption of constant δ_K.

Using the pricing rule (5) and the Fréchet distribution of productivities (4), the distribution of prices within a sector K faced by the representative consumer in location j for goods sourced from location i is:

$$G_{Kjit}(p_K) = \Pr[p_{Kjit} \leq p_K] = 1 - F_{Kit}\left(\frac{w_{it}^{\mu_K} r_{it}^{1-\mu_K} d_{Kji}}{p_K}\right),$$

$$G_{Kjit}(p_K) = 1 - e^{-T_{Kit}\left(w_{it}^{\mu_K} r_{it}^{1-\mu_K} d_{Kji}\right)^{-\theta_K} p_K^{\theta_K}}, \qquad K \in \{A, N\}. \tag{7}$$

Since goods are sourced from the lowest-cost supplier (6), the distribution of prices within sector K in location j for goods actually purchased is:

$$G_{Kjt}\left(p_K\right) = 1 - \prod_{i=1}^{I}\left[1 - G_{Kjit}(p_K)\right] = 1 - e^{-\Phi_{Kjt} p_K^{\theta_K}}, \tag{8}$$

$$\Phi_{Kjt} \equiv \sum_{s=1}^{I} T_{Kst}\left(w_{st}^{\mu_K} r_{st}^{1-\mu_K} d_{Kjs}\right)^{-\theta_K}, \qquad K \in \{A, N\}, \tag{9}$$

where the distribution of prices within sector K in location j for goods actually purchased ($G_{Kjt}\left(p_K\right)$) depends on the productivity parameter (T_{Kit}) for each location, which we calibrate based on the observed employment data, as discussed below.

Expenditure shares across locations: Since goods are sourced from the lowest-cost supplier (6), the probability that location j sources a product h within sector K from location i is:

$$\begin{aligned} \pi_{Kjit} &= \Pr\left[p_{Kjit}\left(h\right) \leq \min\left\{p_{Kjst}\left(h\right)\right\}; s \neq i; K \in \{A, N\}\right] \\ &= \int_0^\infty \prod_{s\neq i}\left[1 - G_{Kjst}\left(p_K\right)\right] dG_{Kjit}\left(p_K\right). \end{aligned}$$

Using the bilateral price distribution (7), the probability that location j sources a product h from location i within sector K can be expressed as:

$$\pi_{Kjit}\left(\mathbf{w_t}, \mathbf{r_t}, \mathbf{T_{Kt}}, \mathbf{d}_K\right) = \frac{T_{Kit}\left(w_{it}^{\mu_K} r_{it}^{1-\mu_K} d_{Kji}\right)^{-\theta_K}}{\sum_{s=1}^{I} T_{Kst}\left(w_{st}^{\mu_K} r_{st}^{1-\mu_K} d_{Kjs}\right)^{-\theta_K}}, \qquad K \in \{A, N\}, \tag{10}$$

where bold math font is used to denote a vector or matrix, so that $\mathbf{T_{Kt}}$ is the vector of productivities across locations in sector K and $\mathbf{d}_K$ is the matrix of bilateral trade costs between locations for sector K.

Following Eaton and Kortum (2002), another implication of the Fréchet distribution of productivities is that the distribution of prices in a given location j for products actually sourced from another location i is independent of the identity of the location i and equal to the distribution of prices in location j. To derive this result, note that the distribution of prices in location j conditional on sourcing products from location i is:

$$\frac{1}{\pi_{Kjit}} \int_0^{p_K} \prod_{s \neq i} \left[1 - G_{Kjst}(q)\right] dG_{Kjit}(q) = 1 - e^{-\Phi_{Kjt} p_K^{\theta_K}} = G_{Kjt}(p_K),$$

where we have used the bilateral and multilateral price distributions, (7) and (8) respectively. Intuitively, under the assumption of a Fréchet productivity distribution, a source location i with a higher scale parameter (T_{Kit}), and hence a higher average productivity, expands on the extensive margin of the number of products supplied exactly to the point at which the distribution of prices for the products it actually sells in destination j is the same as destination j's overall price distribution.

Since the distribution of prices in location j for goods actually purchased is the same across all source locations i, it follows that the share of location j's expenditure within sector K on products sourced from another location i is equal to the probability of sourcing a product from that location (π_{Kjit}). Therefore the share of location j's expenditure within sector K on products sourced from another location i is given by (10).

Expenditure shares across sectors: Using the CES goods consumption index (2), the share of location j's expenditure on goods consumption that is allocated to sector K depends on the dual price index for each sector (P_{Kjt}) as follows:

$$\lambda_{Kjt} = \frac{\psi_K^{\kappa} P_{Kjt}^{1-\kappa}}{\psi_A^{\kappa} P_{Ajt}^{1-\kappa} + \psi_N^{\kappa} P_{Njt}^{1-\kappa}}, \qquad K \in \{A, N\}, \tag{11}$$

where inelastic demand between agriculture and non-agriculture ($0 < \kappa < 1$) implies that a sector's share of goods consumption expenditure is increasing in its relative price index.

Following Eaton and Kortum (2002), the dual price index for each sector (P_{Kjt}) can be determined using the multilateral distribution of prices (8):

$$\begin{aligned} P_{Kjt} &= \left[\int_0^1 p_{Kjt}(h)^{1-\sigma} dh\right]^{\frac{1}{1-\sigma}}, \qquad K \in \{A, N\}, \\ &= \left[\int_0^\infty p_{Kjt}^{1-\sigma} dG_{Kjt}(p_K)\right]^{\frac{1}{1-\sigma}}, \\ &= \left[\int_0^\infty \theta_K \Phi_{Kjt} p_{Kjt}^{\theta_K - \sigma} e^{-\Phi_{Kjt} p_{Kjt}^{\theta_K}} dp_{Kjt}\right]^{\frac{1}{1-\sigma}}. \end{aligned}$$

Using the following change of variable:

$$x_{Kjt} = \Phi_{Kjt} p_{Kjt}^{\theta_K},$$

$$\Rightarrow \qquad p_{Kjt} = \left(\frac{x_{Kjt}}{\Phi_{Kjt}}\right)^{\frac{1}{\theta_K}}, \qquad dp_{Kjt} = \frac{1}{\theta_K}\left(\frac{x_{Kjt}}{\Phi_{Kjt}}\right)^{\frac{1-\theta_K}{\theta_K}} \frac{1}{\Phi_{Kjt}} dx_{Kjt},$$

we obtain:

$$P_{Kjt} = \Phi_{Kjt}^{-1/\theta_K} \left[\int_0^\infty x_{Kjt}^{(1-\sigma)/\theta_K} e^{-x_{Kjt}} dx_{Kjt}\right]^{\frac{1}{1-\sigma}},$$

which implies:

$$P_{Kjt} = \gamma_K \Phi_{Kjt}^{-1/\theta_K} = \gamma_K \left[\sum_{s=1}^{I} T_{Kst}\left(w_{st}^{\mu_K} r_{st}^{1-\mu_K} d_{Kjs}\right)^{-\theta_K}\right]^{-1/\theta_K}, \qquad K \in \{A, N\}, \tag{12}$$

$$\text{where} \qquad \gamma_K \equiv \left[\Gamma\left(\frac{\theta_K + 1 - \sigma}{\theta_K}\right)\right]^{\frac{1}{1-\sigma}},$$

where $\Gamma(\cdot)$ is the gamma function.

Having determined the dual price index for each sector (P_{Kjt}), the dual price index for the aggregate goods consumption index (2) follows immediately:

$$P_{jt} = \left[\psi_A^\kappa P_{Ajt}^{1-\kappa} + \psi_N^\kappa P_{Njt}^{1-\kappa}\right]^{\frac{1}{1-\kappa}}, \tag{13}$$

Combining the sectoral expenditure share (11) with the sectoral price index (12), the share of location j's expenditure on goods consumption that is allocated to sector K can be written solely in terms of trade costs, wages, rents and productivities in all locations:

$$\lambda_{Kjt}\left(\mathbf{w_t}, \mathbf{r_t}, \mathbf{T_{At}}, \mathbf{T_{Nt}}, \mathbf{d}_K\right) = \frac{\psi_K^\kappa \gamma_K^{1-\kappa}\left[\sum_{s=1}^{I} T_{Kst}\left(w_{st}^{\mu_K} r_{st}^{1-\mu_K} d_{Kjs}\right)^{-\theta_K}\right]^{-\frac{1-\kappa}{\theta_K}}}{\sum_{Z\in\{A,N\}} \psi_Z^\kappa \gamma_Z^{1-\kappa}\left[\sum_{s=1}^{I} T_{Zst}\left(w_{st}^{\mu_Z} r_{st}^{1-\mu_Z} d_{Zjs}\right)^{-\theta_Z}\right]^{-\frac{1-\kappa}{\theta_Z}}}. \tag{14}$$

2.4 Factor Markets

Land market clearing in each location implies that total land income equals the sum of payments to residential land plus payments to land used in production in each sector. With a Cobb-Douglas upper tier of utility, a constant fraction of total income is spent on residential land. Additionally, with a Cobb-Douglas production technology, payments to land in each sector and location account for a constant share of revenue, which equals the sum of expenditures on goods produced in that sector and location. Therefore, using the expenditure shares (10) and (11), and noting that total expenditure equals total income, the land market clearing condition can be written as:

$$
\begin{aligned}
r_{it}H_i &= (1-\alpha)\left[w_{it}L_{it} + r_{it}H_i\right] \\
&+ \sum_{K\in\{A,N\}}\sum_{j=1}^{I}\pi_{Kjit}\left(\mathbf{w_t},\mathbf{r_t},\mathbf{T}_{K\mathbf{t}}\right)\lambda_{Kjt}\left(\mathbf{w_t},\mathbf{r_t},\mathbf{T_{At}},\mathbf{T_{Nt}}\right)(1-\mu_K)\,\alpha\left[w_{jt}L_{jt}+r_{jt}H_j\right],
\end{aligned} \tag{15}
$$

where $w_{it}L_{it}$ is the total income of mobile workers; $r_{it}H_i$ is the total income of immobile landlords.

The supply of workers for each location is endogenously determined by population mobility, which requires that the real wage is equalized across all locations populated in equilibrium ($v_{jt} = v_t$ for all t). Using consumer utility (1) and the aggregate price index (13), the real wage in each location can be expressed in terms of the nominal wage (w_{jt}), the price index for each sector (P_{Kjt}) and the nominal land rent (r_{jt}):

$$
v_{jt}\left(\mathbf{w_t},\mathbf{r_t},\mathbf{T_{At}},\mathbf{T_{Nt}},\mathbf{d}_K\right) = v_t = \frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}\eta_{jt}w_{jt}}{\left[\psi_A^{\kappa}P_{Ajt}^{1-\kappa}+\psi_N^{\kappa}P_{Njt}^{1-\kappa}\right]^{\frac{\alpha}{1-\kappa}}r_{jt}^{1-\alpha}},
$$

where the real wage can be written solely in terms of trade costs, wages, rents and productivities using the price index for each sector (12):

$$
v_t = \frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}\eta_{jt}w_{jt}}{\left(\sum_{K\in\{A,N\}}\psi_K^{\kappa}\gamma_K^{1-\kappa}\left[\sum_{s=1}^{I}T_{Kst}\left(w_{st}^{\mu_K}r_{st}^{1-\mu_K}d_{Kjs}\right)^{-\theta_K}\right]^{-\frac{1-\kappa}{\theta_K}}\right)^{\frac{\alpha}{1-\kappa}}r_{jt}^{1-\alpha}}. \tag{16}
$$

Labor market clearing in each location implies that the total income received by the endogenous supply of workers equals the sum of payments to labor across sectors. Following a similar line of reasoning as for land market clearing, the Cobb-Douglas production technology implies that payments to labor in each sector and location account for a constant share of revenue, which equals the sum of expenditures on goods produced in that sector and location. Therefore, using the expenditure shares (10) and (11), and noting that total expenditure equals total income, the labor market clearing condition can be written as:

$$
w_{it}L_{it} = \sum_{K\in\{A,N\}}\sum_{j=1}^{I}\pi_{Kjit}\left(\mathbf{w_t},\mathbf{r_t},\mathbf{T_{Kt}}\right)\lambda_{Kjt}\left(\mathbf{w_t},\mathbf{r_t},\mathbf{T_{At}},\mathbf{T_{Nt}}\right)\mu_K\alpha\left[w_{jt}L_{jt}+r_{jt}H_j\right]. \tag{17}
$$

2.5 General Equilibrium

Given productivity in each sector and location $\{T_{Ait}, T_{Nit}\}$, the model can be solved by determining wages, rents and labor supplies for each location $\{w_{it},\ r_{it},\ L_{it}\}$, which can be used to determine all other endogenous variables of the model, including employment for each sector and location $\{L_{Ait},\ L_{Nit}\}$. The equilibrium vector of wages, rents and labor supplies is determined by the system of three equations for all locations given by labor market clearing (17), land market clearing (15) and population mobility (16).

In our quantitative analysis, rather than solving for the equilibrium vector given productivity, we instead assume that observed employment in each sector and location $\{L_{Ait},\ L_{Nit}\}$ is an equilibrium of the model and calibrate the productivity in each sector and location $\{T_{Ait}, T_{Nit}\}$ required to support observed employment as an equilibrium. In addition to determining productivity, the calibration also yields solutions for wages, rents and the other endogenous variables of the model.

2.6 Model Calibration

The calibration of the model uses the expressions for labor payments in each sector from the labor market clearing condition (17), the land market clearing condition (15), the population mobility condition (16), observed employment in each sector and location, land area and bilateral distance $\{L_{Ait},\ L_{Nit},\ H_i,\ \tau_{ij}\}$, to calibrate unobserved productivity in each sector and location $\{T_{Ait},\ T_{Nit}\}$.

The remainder of this section proceeds as follows. In Subsection 2.6.1, we solve for land rents in terms of wages and observables. In Subsection 2.6.2, we use the expression for labor payments in each sector to derive a first relationship between productivity in each sector and wages. In Subsection 2.6.3, we use population mobility across locations to derive a second relationship between wages and productivity in each sector. In Subsection 2.6.4, we discuss how these two relationships can be used to calibrate unobserved productivity for each sector and location $\{T_{Ait},\ T_{Nit}\}$.

2.6.1 Land Rents

We begin by combining labor payments in each sector and land market clearing to obtain an expression for land rents in terms of wages and observables. Note that labor payments in a given sector and location account for a constant fraction of revenue, which equals total expenditure on goods produced in that sector and location. Using the Cobb-Douglas upper tier of utility, the share of goods consumption expenditure allocated to each sector (λ_{Kit}), the share of sectoral expenditure allocated to each location (π_{Kjit}), and the Cobb-Douglas production technology, labor payments in a given sector and location can be written as:

$$w_{it}L_{Kit} = \sum_{i=1}^{I} \pi_{Kjit}\left(\mathbf{w_t}, \mathbf{r_t}, \mathbf{T_{Kt}}, \mathbf{d}_K\right) \lambda_{Kjt}\left(\mathbf{w_t}, \mathbf{r_t}, \mathbf{T_{At}}, \mathbf{T_{Nt}}, \mathbf{d}_K\right) \mu_K \alpha \left[w_{jt}L_{jt} + r_{jt}H_j\right], \quad (18)$$

where bold math font again denotes a vector.

Using this expression for labor payments in the land market clearing condition (15), land rents (r_{it}) can be solved for as a function of observed employment (L_{Ait} and L_{Nit}), observed land area (H_i), wages (w_{it}) and parameters:

$$r_{it} = \vartheta_{it}w_{it}, \qquad \vartheta_{it} \equiv \frac{1}{\alpha}\frac{L_{it}}{H_i}\left[(1-\alpha) + \left(\frac{1-\mu_A}{\mu_A}\right)\frac{L_{Ait}}{L_{it}} + \left(\frac{1-\mu_N}{\mu_N}\right)\frac{L_{Nit}}{L_{it}}\right], \quad (19)$$

where $L_{it} = L_{Ait} + L_{Nit}$. In this expression, the relative value of rents and wages (ϑ_{it}) is pinned down by parameters and observed data $\{L_{Ait},\ L_{Nit},\ H_i\}$.

Therefore, equilibrium relative factor prices (r_{it}/w_{it}) depend through ϑ_{it} on the share of residential land in consumer expenditure $(1-\alpha)$, the observed ratio of total employment to land area (L_{it}/H_i), factor intensities in each sector (μ_K), and the observed share of each sector in total employment (L_{Kit}/L_{it}).

2.6.2 Labor Payments

Having solved for land rents as a function of wages, we now show how labor payments for each sector can be used to determine productivity in each sector given wages. Substituting for land rents (r_{it}) using (19), payments to labor (18) in each sector and location can be written as the following implicit function:

$$\Lambda_{Kit} = w_{it}L_{Kit} - \sum_{j=1}^{I} \pi_{Kjit}\left(\mathbf{w_t}, \boldsymbol{\vartheta}_\mathbf{t}, \mathbf{T}_{K\mathbf{t}}, \mathbf{d}_K\right) \lambda_{Kjt}\left(\mathbf{w_t}, \boldsymbol{\vartheta}_\mathbf{t}, \mathbf{T_{At}}, \mathbf{T_{Nt}}, \mathbf{d}_K\right) \mu_K \alpha w_{jt} \left[L_{jt} + \vartheta_{jt}H_j\right] = 0, \quad (20)$$

where $\boldsymbol{\vartheta}_\mathbf{t}$ is determined by observables.

Proposition 1 *Given observed employment (L_{Kit}), land area (H_i) and bilateral distance (τ_{ji} and hence d_{Kji}) and assumed values of wages (w_{it}), there exists a unique (normalized) value for calibrated productivity for each sector and location (T_{Kit}) that solves the labor payments condition (20) for each sector and location.*

Proof. See Subsection 2.9 of this web appendix. ∎

Intuitively, for given assumed values of wages, observed employment for each sector and location determines the value that productivity must take for each sector and location in an equilibrium where the total value of labor demand equals the total value of labor supply.

2.6.3 Population Mobility

We now derive another relationship between wages and productivity from the population mobility condition, which requires workers to receive the same equilibrium real income across all populated locations. Substituting for land rents (r_{it}) using (19), the population mobility condition (16) can be re-written as the following implicit function:

$$\Omega_{it} = \frac{\left[\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\eta_{jt}\right]^{1/\alpha} w_{jt}}{\left(\sum_{K\in\{A,N\}}\psi_K^{\kappa}\gamma_K^{1-\kappa}\left[\sum_{s=1}^{I}T_{Kst}\left(w_{st}\vartheta_{st}^{1-\mu_K}d_{Kjs}\right)^{-\theta_K}\right]^{-\frac{1-\kappa}{\theta_K}}\right)^{\frac{1}{1-\kappa}}\vartheta_{jt}^{(1-\alpha)/\alpha}} - \frac{\left[\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\eta_{it}\right]^{1/\alpha} w_{it}}{\left(\sum_{K\in\{A,N\}}\psi_K^{\kappa}\gamma_K^{1-\kappa}\left[\sum_{s=1}^{I}T_{Kst}\left(w_{st}\vartheta_{st}^{1-\mu_K}d_{Kis}\right)^{-\theta_K}\right]^{-\frac{1-\kappa}{\theta_K}}\right)^{\frac{1}{1-\kappa}}\vartheta_{it}^{(1-\alpha)/\alpha}} = 0, \quad (21)$$

for all populated locations i and j, where ϑ_{it} is determined by observables.

Proposition 2 *Given observed employment (L_{Kit}), land area (H_i) and bilateral distance (τ_{ji} and hence d_{Kji}) and assumed values of productivities (T_{Ait}, T_{Nit}), there exists a unique (normalized) wage for each location (w_{it}) that solves the population mobility condition (21) for each sector and location.*

Proof. See Subsection 2.10 at the end of this appendix. ∎

Intuitively, real wages in each location depend on wages, consumer prices and rents. But rents can be solved for as a function of wages and observables, which in turn implies that consumer price indices can be determined as a function of wages and productivities. Therefore, given assumed values of productivities in each sector and location, we can solve for the unique values of wages for each location for which real wages are equalized.

2.6.4 Productivity and Wages

The calibration of the model is fully described by the labor payments condition (20) for the two sectors and the population mobility condition (21). Together these provide a system of three equations for each location that determines productivity for the two sectors and wages for each location $\{T_{Ait}, T_{Nit}, w_{it}\}$, given observed employment in each sector and location, land area and distance.

To calibrate the model, we first assume values of wages for each location and use the labor payments condition (20) for the two sectors to determine unique values of productivity for each sector and location $\{T_{Ait}, T_{Nit}\}$. We next use these calibrated productivities and the population mobility condition (21) to determine unique values of wages for each location $\{w_{it}\}$. If these solutions for wages differ from the assumed values for wages used to calibrate productivity, we update the assumed values of wages and re-calibrate productivities for each sector and location. We repeat this process until the solutions for wages from the population mobility condition equal the assumed values of wages used in the labor payments conditions. For different initial assumed values of wages, we find that the system of equations given by the labor payments and population mobility conditions (20)-(21) converges to a unique equilibrium. Using the resulting solutions for equilibrium wages and productivity in the two sectors $\{w_{jt}, T_{Ajt}, T_{Njt}\}$, all other endogenous variables of the model can be determined.

From the labor payments and population mobility conditions (20)-(21), differences in consumption amenities $\{\eta_{it}\}$ across locations have similar effects on equilibrium wages as differences in productivities $\{T_{Ait}, T_{Nit}\}$ across locations that are common to both sectors.[10] Therefore we normalize $\alpha^{\alpha}(1-\alpha)^{1-\alpha}\eta_{it} = 1$, so that differences in consumption amenities across locations are captured in the calibrated productivities.

The labor payments conditions for the two sectors (20) are homogeneous of degree zero in productivities $\{T_{Ait}, T_{Nit}\}$, which reflects the fact that in general equilibrium the distribution of employment across sectors and locations depends on relative rather than absolute levels of productivity. To calibrate productivity, we impose the following two normalizations. First, we decompose productivity in each sector K, location i and year t (T_{Kit}) into an aggregate component that is common across locations (T^{S}_{Kt}) and an idiosyncratic component that is

[10] In the presence of bilateral trade costs, higher productivity in a location reduces its consumer price index and hence raises its indirect utility relative to other locations through the denominator in (21). In contrast, greater consumption amenities in a location raise its indirect utility relative to other locations through the numerator in (21).

specific to each location (T^I_{Kit}): $T_{Kit} = T^S_{Kt} T^I_{Kit}$. We normalize idiosyncratic productivity to have a mean of one hundred across locations in each sector and year, $\frac{1}{I}\sum_{i=1}^{I} T^I_{Kit} = 100$ for $K \in \{A, N\}$. With this first normalization, T^I_{Kit} captures a location's productivity in a sector and year relative to average productivity in that sector and year, while $\{T^S_{At}, T^S_{Nt}\}$ capture average levels of relative productivity in the two sectors. Second, we normalize aggregate productivity in non-agriculture to equal one hundred ($T^S_{Nt} = 100$). With this second normalization, T^S_{At} captures aggregate productivity in agriculture relative to non-agriculture in each year. Although the use of alternative normalizations results in different absolute levels of productivities, it leaves relative productivities unchanged.[11]

2.7 Model Parameters

Our calibration of the model requires data on employment by sector and location, which is not available for MCDs for intermediate years between 1880 and 2000. To examine the timing of structural transformation across these intermediate years, we calibrate the model using our counties dataset for twenty-year intervals from 1880-2000. To compare the results of the model calibration with our baseline empirical findings in Michaels et al. (2011), we concentrate on our baseline sample of the A and B states, which have a longer history of settlement and hence are likely to be more closely approximated by the assumption that observed employment is an equilibrium of the model. Nonetheless, we find a similar pattern of results for the full sample of states.

We choose values for the model's parameters based on central estimates from the existing empirical literature. Using the estimates for the United States in Davis and Ortalo-Magne (2008), we set of the share of residential land in consumer expenditure $(1 - \alpha)$ equal to 0.25. Following a large literature in macroeconomics, including Ngai and Pissarides (2007), we assume inelastic demand between agriculture and non-agriculture, and set the elasticity of substitution across the two sectors (κ) equal to 0.5. Within each sector, we assume that different products are substitutes for one another and set the elasticity of substitution within sectors (σ) equal to 4, which is comparable to the empirical estimate for U.S. manufacturing plants of 3.8 in Bernard, Eaton, Jensen and Kortum (2003).

[11] Although improvements in productivity that are common across sectors and locations leave the distribution of employment across sectors and locations unchanged, they do increase indirect utility. Under our two normalizations, the common absolute level of productivity across sectors and locations is captured in the common level of indirect utility across all locations (v_t).

To focus on differences in average productivity across sectors and locations (T_{Ajt}, T_{Njt}), we assume that the Fréchet shape parameter (θ_K) determining the dispersion of productivities is the same across sectors. Following the trade literature, we assume a value of θ_K equal to 4, which is consistent with the estimates in Donaldson (2010), Eaton and Kortum (2002) and Simonovska and Waugh (2011). For the elasticity of trade costs with respect to distance (δ_K), we assume a value of 0.33, which is in line with the range of estimates in Limao and Venables (2001) and Hummels (2007). Together the elasticity of trade costs with respect to distance of 0.33 and the Fréchet shape parameter of 4 imply an elasticity of trade flows with respect to distance ($-\delta_K \theta_K$) of -1.33, which is consistent with the empirical estimates from the gravity equation literature surveyed in Disdier and Head (2008). To measure transport costs between pairs of counties, we use bilateral great circle distances between their centroids. To measure the transport costs that a county incurs in serving itself, we follow the empirical economic geography literature in constructing a measure of a county's internal distance. Following Head and Mayer (2004) and Redding and Venables (2004), we approximate each county with a circle that has the same area as the county and evaluate each county's internal distance as the average distance between pairs of points within that circle.

In the model, there is a one-to-one relationship between the shares of labor and land in production costs and the shares of mobile and immobile factors in production costs. In the data, the mapping from mobile and immobile factors to labor and land may be imperfect, because there are additional factors of production other than labor and land (e.g. physical capital), and because other factors of production besides land can exhibit a degree of immobility across locations (e.g. buildings and structures). To determine the share of immobile factors in production costs, we use the U.S. estimates from Table 5 of Valentinyi and Herrendorf (2008). For non-agriculture, we set the share of immobile factors in costs ($1-\mu_N$) equal to 0.18. For agriculture, we set the share of immobile factors in costs ($1-\mu_A$) equal to 0.22.

2.8 Quantitative Results

Our calibration of the model is undertaken for each year separately without imposing prior structure on the pattern of productivity growth over time. In this subsection, we examine whether the resulting calibrated productivities are consistent with our explanation for the stylized facts based on aggregate reallocation and differences in mean reversion across sectors.

Before examining the calibrated productivities, we present evidence on the pattern of aggregate reallocation away from agriculture over time. In Panel A of Figure 1, we display agriculture's share of aggregate employment in our counties dataset for each year of our sample. As apparent from the figure, there is a substantial aggregate reallocation of employment away from agriculture, which decelerates from 1920-1940 (during the agricultural depression of the 1920s and the Great Depression of the 1930s) and accelerates from 1940-1960 (in the years surrounding the Second World War). These changes in the pace of aggregate reallocation are also evident in Panel B of Figure 1, which shows the annualized proportional rate of growth in agriculture's share of employment for each twenty-year period. By the end of our sample, the agricultural sector accounts for less than two percent of aggregate employment (Panel A), but its small size is associated with large proportional rates of decline in its share of aggregate employment (Panel B).

In Panel C of Figure 1, we show the annualized proportional rate of growth of calibrated aggregate productivity in agriculture relative to non-agriculture for each twenty-year period ($\Delta \ln \left(T_{At}^{S}/T_{Nt}^{S}\right)$). Care should be taken in interpreting these rates of growth of calibrated relative aggregate productivity, because they are based on our normalization that the relative weight of sectors in demand is constant over time. Under this normalization, the rates of growth of calibrated relative aggregate productivity capture both true differences in aggregate productivity growth across sectors and changes in the common relative weight of sectors in demand (e.g. due to non-homothetic preferences or changes in product quality). Despite these caveats, the finding of faster aggregate productivity growth in agriculture than in non-agriculture in Panel C is consistent with the results of empirical growth accounting studies for the U.S. during our sample period, including Kuznets (1966) from 1870-1940 and Maddison (1980) from 1950-1976. Furthermore, these findings are also consistent with the economic history literature on the development of U.S. agriculture, which emphasizes the role of productivity-enhancing improvements in technology, such as mechanization and biological innovation (see Cochrane 1979 and Rasmussen 1962).

The constant elasticity functional forms in the model imply that proportional changes in relative aggregate productivity across sectors are associated with proportional changes in the shares of sectors in aggregate employment. Therefore the pattern of relative aggregate productivity growth in Panel C closely mirrors the pattern of proportional changes in agriculture's share of aggregate employment in Panel B. The relationship, however, is not necessarily one-for-one, because the calibrated values of aggregate productivity depend on the distribution of employment across sectors and locations with different idiosyncratic productivities and bilateral transport costs. As a result, a given change in relative aggregate productivity can be associated with different changes in agriculture's share of aggregate employment depending on the pattern of idiosyncratic productivity growth and employment across locations in each sector.

Our explanation for the stylized facts combines the above aggregate reallocation away from agriculture with a sharp decline in the initial share of agriculture in a location's employment at intermediate initial population densities (Stylized Fact 3). In Panel D of Figure 1, we examine the source of these differences in initial agricultural specialization in the model by displaying initial log idiosyncratic productivity in non-agriculture relative to agriculture against initial log population density. The estimated coefficient from the OLS regression relationship shown in the figure is 1.107 (with standard error 0.088) and the R^2 is 0.31. At intermediate initial population densities, there is a sharp rise in productivity in non-agriculture relative to agriculture, which reduces production costs in non-agriculture relative to agriculture, and contributes towards the sharp decline in agriculture's share of employment at these densities. Since agriculture is more land intensive than non-agriculture, and more densely-populated locations are characterized by higher equilibrium land rents, this difference in factor intensity across sectors is another force that contributes towards the decline in agriculture's share of employment at intermediate initial population densities.

In Panels E and F of Figure 1, we examine the relationship between idiosyncratic productivity growth and initial log idiosyncratic productivity in each sector. From the x-axes in Panels E and F, the calibrated log 1880 productivities feature greater dispersion in non-agriculture than in agriculture, which is consistent with our empirical findings of greater dispersion in employment density in non-agriculture than in agriculture (Stylized Fact 4).[12]

Comparing Panels E and F, the calibrated idiosyncratic productivities exhibit greater mean reversion in agriculture than in non-agriculture (Stylized Facts 5 and 6). Regressing idiosyncratic productivity growth on initial log idiosyncratic productivity, the estimated coefficient for agriculture is negative and statistically significant (-0.0027 with standard error 0.0003), whereas the estimated coefficient for non-agriculture is statistically insignificant and

[12] Although there are a small number of counties with high 1880 agricultural productivity in Panel E, which reflects positive agricultural employment in a few counties with high population densities, 1880 productivity is less dispersed in Panel E for agriculture than in Panel F for non-agriculture.

its absolute value is about an order of magnitude smaller (-0.0002 with standard error 0.0003). This evidence of mean reversion in agricultural productivity growth is consistent with the historical literature on the development of U.S. agriculture (see for example Cochrane 1979). As discussed by Olmstead and Rhode (2002) for the case of wheat, a number of the productivity-enhancing improvements in agricultural technology during our sample period favored areas with poorer climate and soil. Since poor climate and soil are reflected in low initial levels of agricultural productivity in the model, technological improvements that raise the relative productivity of areas with poorer climate and soil generate mean reversion in agricultural productivity.[13]

In calibrating productivity for each sector and location, we use the structure of the model together with observed employment, land area and bilateral distance. The results so far provide support for our explanation for the stylized facts based on a combination of aggregate reallocation and differences in mean reversion in productivity growth across sectors. As an additional check on the model's explanatory power, we now examine its predictions for variables not used in its calibration. Here data availability constrains our analysis to the year 2000 for which we have county data on other economic outcomes.

In Panel A of Figure 2, we display log land rents in the model in 2000 against log median house prices in the data in 2000.[14] Although there are several reasons why median house prices can differ from land rents, and why the difference between them can change across locations, there is a strong relationship between them. Regressing log land rents in the model on log median house prices in the data, we find a statistically significant positive coefficient of 3.518 (with standard error 0.161). This coefficient of greater than one is consistent with the idea that housing prices are less variable than land rents, because land is only one component of housing costs, and higher land rents can be partially offset by lower land use through higher housing density. From the regression R^2 of 0.52, the model has substantial explanatory power for the observed variation in house prices.

In Panel B of Figure 2, we display log wages in the model in 2000 against log per capita wages in the data in 2000. Although the model abstracts from many idiosyncratic factors that can affect wages in individual counties, there is a strong positive relationship between

[13] Using farm output per kilometer squared as a crude measure of agricultural productivity, we find evidence of mean reversion in agricultural productivity in our county sub-periods dataset, as discussed in Michaels et al. (2011).

[14] For further details on the measures of house prices and wages used in this subsection, see Michaels et. al (2011).

the model's predictions and the data. Regressing log wages in the model on log wages in the data, we find a statistically significant coefficient of 1.184 (with standard error 0.039). This estimated coefficient of around one suggests a close correspondence between the model's predictions and the data, although we can reject a coefficient of exactly one at conventional levels of statistical significance. The regression R^2 of 0.55 confirms that the model also has substantial explanatory power for the observed variation in wages.

Within the model, the population mobility condition implies that the higher land rents of more densely-populated locations must be offset in equilibrium by either higher wages, lower consumer goods price indices, or higher amenities. Variation in consumer goods price indices arises from bilateral transport costs, which generate spatial interactions between locations in product markets depending on their geographical orientation relative to one another. Using the population mobility condition (16) and our normalization of consumption amenities $\left(\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\eta_{jt}=1\right)$, the variation in log weighted rents $\left(\log\left(r_{jt}^{1-\alpha}\right)\right)$ can be decomposed into the variation in log wages $(\log w_{jt})$ and log weighted consumer goods price indices $\left(\log\left(P_{jt}^{\alpha}\right)\right)$:

$$\log\left(r_{jt}^{1-\alpha}\right)=\log w_{jt}-\log\left(P_{jt}^{\alpha}\right)-\log v_t, \tag{22}$$

where v_t is constant across locations j within a given year t.

Running separate OLS regressions of $\log w_{jt}$ and $-\log\left(P_{jt}^{\alpha}\right)$ on $\log\left(r_{jt}^{1-\alpha}\right)$, the coefficients on $\log w_{jt}$ and $-\log\left(P_{jt}^{\alpha}\right)$ sum to one across these two separate regressions and capture the average contributions of wages and weighted consumer goods price indices to variation in weighted rents across locations. In Panel C of Figure 2, we show the results of this regression decomposition for the year 2000 by displaying the log weighted consumer goods price index against log weighted rents and the OLS regression relationship between them. We find that a one percent increase in rents is associated on average with a 0.85 percent increase in wages and a 0.15 percent reduction in consumer goods prices. Intuitively, the higher land rents in more densely-populated locations are compensated in equilibrium by higher wages, which requires higher productivity. This higher productivity not only rationalizes the higher wages, but also increases the supply of locally-produced goods, which are available at low transport costs, and hence reduces consumer goods price indices.[15]

[15] The lower consumer goods price index in more densely-populated locations is reminiscent of the "price index effect" in new economic geography models (see Fujita et al. 1999), but here the range of goods available for consumption is fixed. Using U.S. scanner data, Handbury and Weinstein (2010) find lower prices for a product with a given barcode in a given type of store in more densely-populated locations.

Higher productivity in a more densely-populated location not only reduces the consumer goods price index in that location, but also reduces the consumer goods price index in neighboring locations, which benefit from the resulting increase in the supply of locally-produced goods. In Panel D of Figure 2, we provide evidence on the magnitude of these spatial interactions between locations by graphing the log consumer goods price index in the model against the log distance-weighted sum of employment in all other locations in the data. This distance-weighted sum is sometimes referred to as "market potential" and is calculated here excluding the own location. As apparent from the figure, counties close to concentrations of employment in other counties experience lower consumer price indices as a result of the increased supply of locally-available goods, where the OLS regression line shown in the figure has an estimated coefficient of -0.064 (with standard error 0.002).

Therefore the results from the calibrated model confirm the explanation for the stylized facts in Michaels et al. (2011) based on a combination of an aggregate reallocation from agriculture to non-agriculture and differences in mean reversion in productivity between the two sectors. The results from the calibrated model also generate predictions that are consistent with other data not used in the calibration, and are informative for the spatial equilibrium relationship between wages, rents and consumer prices across locations.

2.9 Proof of Proposition 1

Proof. The labor payments condition has the following properties:
(i) $\mathbf{\Lambda_{Kt}}\left(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}}\right)$ is continuous in $\{\mathbf{T_{Kt}},\ \mathbf{T_{-Kt}}\}$ from inspection of (20).
(ii) $\mathbf{\Lambda_{Kt}}\left(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}}\right)$ is homogenous of degree zero in $\{\mathbf{T_{Kt}},\ \mathbf{T_{-Kt}}\}$, since increasing both $\mathbf{T_{Kt}}$ and $\mathbf{T_{-Kt}}$ by a constant proportional amount $\varrho > 1$ for each location leaves $\{\pi_{Kjit}$, π_{-Kjit}, λ_{Kjt}, $\lambda_{-Kjt}\}$ unchanged in (20).
(iii) $\sum_{i=1}^{I} \Lambda_{Kit}\left(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}}\right) = 0$ since:

$$\sum_{i=1}^{I} \Lambda_{Kit} = \sum_{i=1}^{I} w_{it} L_{Kit} - \mu_K \sum_{j=1}^{I} \lambda_{Kjt}\left(\mathbf{w_t}, \boldsymbol{\vartheta}_\mathbf{t}, \mathbf{T_{At}}, \mathbf{T_{Nt}}, \mathbf{d}_K\right) \alpha w_{jt}\left[L_{jt} + \vartheta_{jt} H_j\right] = 0,$$

where the first term is total labor income in sector K and the second term is total payments to labor from expenditure in sector K.
(iv) $\mathbf{\Lambda_{Kt}}\left(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}}\right)$ exhibits gross substitution in $\mathbf{T_{Kt}}$ given the assumed values of wages $(\mathbf{w_t})$ as long as the share of each location in sectoral expenditure (π_{Kjit}) is sufficiently small.

Consider first the effect on Λ_{Kit} of an increase in location i's productivity within sector K:

$$\frac{d\Lambda_{Kit}}{dT_{Kit}} = -\sum_{j=1}^{I} \frac{d\pi_{Kjit}}{dT_{Kit}} \lambda_{Kjt}\mu_K \alpha w_{jt} \left[L_{jt} + \vartheta_{jt} H_j\right] - \sum_{j=1}^{I} \pi_{Kjit} \frac{d\lambda_{Kjt}}{dT_{Kit}} \mu_K \alpha w_{jt} \left[L_{jt} + \vartheta_{jt} H_j\right],$$

$$\frac{d\Lambda_{Kit}}{dT_{Kit}} = -\sum_{j=1}^{I} \frac{1}{T_{Kit}} \left\{1 - \pi_{Kjit}\left[1 + (1-\lambda_{Kjt})\left(\frac{1-\kappa}{\theta_K}\right)\right]\right\} \pi_{Kjit}\lambda_{Kjt}\mu_K \alpha w_{jt}\left[L_{jt} + \vartheta_{jt} H_j\right] < 0, \tag{23}$$

for π_{Kjit} sufficiently small, since $\theta_K > 1 > 1 - \kappa > 0$ and $0 < 1 - \lambda_{Kjt} < 1$.

On the one hand, $d\pi_{Kjit}/dT_{Kit} > 0$, since higher productivity in a location in a given sector raises its share of expenditure within that sector, which increases expenditure on the location's products within that sector. On the other hand, $d\lambda_{Kjt}/dT_{Kit} < 0$, since higher productivity in a location in a given sector reduces the price index for that sector, which with inelastic demand reduces the share of overall expenditure on that sector, which in turn reduces expenditure on the location's products within that sector. As long as the share of each location in sectoral expenditure (π_{Kjit}) is sufficiently small, as is satisfied in our data for these broad sectors of agriculture and non-agriculture, the effect of an individual location's higher productivity on the sectoral price index is small. Therefore the first effect dominates, so that expenditure on a location's products within a sector is increasing in its productivity within that sector given the assumed values of wages.

Consider next the effect on Λ_{Kit} of an increase in another location s's productivity within sector K:

$$\frac{d\Lambda_{Kit}}{dT_{Kst}} = -\sum_{j=1}^{I} \frac{d\pi_{Kjit}}{dT_{Kst}} \lambda_{Kjt}\mu_K \alpha w_{jt} \left[L_{jt} + \vartheta_{jt} H_j\right] - \sum_{j=1}^{I} \pi_{Kjit} \frac{d\lambda_{Kjt}}{dT_{Kst}} \mu_K \alpha w_{jt} \left[L_{jt} + \vartheta_{jt} H_j\right], \quad s \neq i,$$

$$\frac{d\Lambda_{Kit}}{dT_{Kst}} = \sum_{j=1}^{I} \frac{\pi_{Kjst}}{T_{Kst}} \left[1 + (1-\lambda_{Kjt})\left(\frac{1-\kappa}{\theta_K}\right)\right] \pi_{Kjit}\lambda_{Kjt}\mu_K \alpha w_{jt}\left[L_{jt} + \vartheta_{jt} H_j\right] > 0, \quad s \neq i, \tag{24}$$

where $\theta_K > 1 > 1 - \kappa > 0$ and $0 < 1 - \lambda_{Kjt} < 1$.

Additionally Λ_{Kit} is monotone in the productivity of each location in the other sector $-K$:

$$\frac{d\Lambda_{Kit}}{dT_{-Kst}} = -\sum_{j=1}^{I} \pi_{Kjit} \frac{d\lambda_{Kjt}}{dT_{-Kst}} \mu_K \alpha w_{jt} \left[L_{jt} + \vartheta_{jt} H_j\right],$$

$$\frac{d\Lambda_{Kit}}{dT_{-Kst}} = -\sum_{j=1}^{I} \frac{\pi_{-Kjst}\lambda_{-Kjt}}{T_{-Kst}} \left(\frac{1-\kappa}{\theta_{-K}}\right) \pi_{Kjit}\lambda_{Kjt}\mu_K \alpha w_{jt}\left[L_{jt} + \vartheta_{jt} H_j\right] < 0, \quad \forall s. \tag{25}$$

Note that $d\lambda_{Kjt}/dT_{-Kit} > 0$ since higher productivity in another location in sector $-K$ (where we use $-K$ to indicate the other sector, not sector K) reduces the price index for sector $-K$, which with inelastic demand increases the share of expenditure on sector K, which in turn increases expenditure on each location's products within sector K.

Since $\mathbf{\Lambda_{Kt}}(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}})$ exhibits gross substitution in $\mathbf{T_{Kt}}$ and is monotone in $\mathbf{T_{-Kt}}$, this system of equations has at most one (normalized) solution. Gross substitution implies that $\mathbf{\Lambda_{Kt}}(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}}) = \mathbf{\Lambda_{Kt}}(\mathbf{T'_{Kt}}, \mathbf{T_{-Kt}})$ cannot occur whenever $\mathbf{T_{Kt}}$ and $\mathbf{T'_{Kt}}$ are two productivity vectors that are not colinear. Since $\mathbf{\Lambda_{Kt}}(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}})$ is homogenous of degree zero in $\{\mathbf{T_{Kt}}, \mathbf{T_{-Kt}}\}$, we can assume $\mathbf{T'_{Kt}} \geq \mathbf{T_{Kt}}$ and $T_{Kit} = T'_{Kit}$ for some i. Now consider altering the productivity vector $\mathbf{T'_{Kt}}$ to obtain the productivity vector $\mathbf{T_{Kt}}$ in $I-1$ steps, lowering (or keeping unaltered) the productivities of all the other $I-1$ locations $s \neq i$ one at a time. By gross substitution, $\Lambda_{Kit}(\cdot)$ cannot decrease in any step, and because $\mathbf{T_{Kt}} \neq \mathbf{T'_{Kt}}$, it will actually increase in at least one step. Hence $\mathbf{\Lambda_{Kt}}(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}}) > \mathbf{\Lambda_{Kt}}(\mathbf{T'_{Kt}}, \mathbf{T_{-Kt}})$ and we have a contradiction.

We next establish that there exists a productivity vector in each sector, $\{\mathbf{T^*_{Kt}}, \mathbf{T^*_{-Kt}}\} \in \Re^I_+$, such that $\mathbf{\Lambda_{Kt}}(\mathbf{T^*_{Kt}}, \mathbf{T^*_{-Kt}}) = \mathbf{\Lambda_{-Kt}}(\mathbf{T^*_{Kt}}, \mathbf{T^*_{-Kt}}) = 0$. By homogeneity of degree zero, we can restrict our search for this productivity vector for each sector to the unit simplex: $\Delta = \left\{\mathbf{T_{Kt}} \in \Re^I_+ : \sum_{i=1}^I T_{Kit} = 1\right\}$. Define on Δ the function $\Lambda^+_{Kit}(\cdot)$ by $\Lambda^+_{Kit}(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}}) = \max\{\Lambda_{Kit}(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}}), 0\}$. Note that $\Lambda^+_{Kit}(\cdot)$ is continuous. Denote $\varpi_{Kt}(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}}) = \sum_{i=1}^I \left[T_{Kit} + \Lambda^+_{Kit}(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}})\right]$. We have $\varpi_{Kt}(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}}) \geq 1$ for all $\{\mathbf{T^*_{Kt}}, \mathbf{T^*_{-Kt}}\} \in \Re^I_+$. Define a continuous function $\boldsymbol{\varsigma}_{\mathbf{Kt}}(\cdot)$ from the closed convex set Δ into itself by:

$$\boldsymbol{\varsigma}_{\mathbf{Kt}}(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}}) = [1/\varpi_{Kt}(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}})]\left[\mathbf{T_{Kt}} + \mathbf{\Lambda^+_{Kt}}(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}})\right].$$

Note that this fixed-point function tends to increase T_{Kit} for locations with $\Lambda_{Kit}(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}}) > 0$. By Brouwer's Fixed-point Theorem, there exists $\mathbf{T^*_{Kt}} \in \Delta$ for each sector K such that $\mathbf{T^*_{Kt}} = \boldsymbol{\varsigma}_{\mathbf{Kt}}(\mathbf{T^*_{Kt}}, \mathbf{T^*_{-Kt}})$ and $\mathbf{T^*_{-Kt}} = \boldsymbol{\varsigma}_{\mathbf{-Kt}}(\mathbf{T^*_{Kt}}, \mathbf{T^*_{-Kt}})$.

Since $\sum_{i=1}^I \Lambda_{Kit}(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}}) = 0$, it cannot be the case that $\Lambda_{Kit}(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}}) > 0$ for all $i = 1, \ldots, I$ or $\Lambda_{Kit}(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}}) < 0$ for all $i = 1, \ldots, I$. Additionally, if $\Lambda_{Kit}(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}}) > 0$ for some i and $\Lambda_{Kst}(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}}) < 0$ for some $s \neq i$, $\mathbf{T^*_{Kt}} \neq \boldsymbol{\varsigma}_{\mathbf{Kt}}(\mathbf{T^*_{Kt}}, \mathbf{T^*_{-Kt}})$. It follows that at the fixed point for wages, $\mathbf{T^*_{Kt}} = \boldsymbol{\varsigma}_{\mathbf{Kt}}(\mathbf{T^*_{Kt}}, \mathbf{T^*_{-Kt}})$, $\Lambda_{Kit}(\mathbf{T_{Kt}}, \mathbf{T_{-Kt}}) = 0$ for all i. ■

2.10 Proof of Proposition 2

Proof. The population mobility condition (21) has the following properties:
(i) $\Omega_{it}(\mathbf{w_t})$ is continuous in $\{\mathbf{w_t}\}$ from inspection of (21).
(ii) $\Omega_{it}(\mathbf{w_t})$ is homogeneous of degree zero in $\{\mathbf{w_t}\}$, since increasing w_{it} by a constant proportional amount $\varrho > 1$ for each location i increases both the numerator and denominator of (21) by the proportion ϱ.
(iii) $\sum_{i=1}^{I}\Omega_{it}(\mathbf{w_t}) = 0$ from inspection of (21).
(iv) $\Omega_{it}(\mathbf{w_t})$ exhibits gross substitution in $\mathbf{w_t}$ given the assumed productivities $(\mathbf{T_{At}}, \mathbf{T_{Nt}})$:

$$\frac{d\Omega_{it}}{dw_{it}} = -\left[1 + \sum_{K\in\{A,N\}}\pi_{Kjit}\lambda_{Kjt} - \sum_{K\in\{A,N\}}\pi_{Kiit}\lambda_{Kit}\right]\frac{v_t^{1/\alpha}}{w_{it}} < 0,$$

$$\frac{d\Omega_{it}}{dw_{jt}} = \left[1 + \sum_{K\in\{A,N\}}\pi_{Kijt}\lambda_{Kit} - \sum_{K\in\{A,N\}}\pi_{Kjjt}\lambda_{Kjt}\right]\frac{v_t^{1/\alpha}}{w_{jt}} > 0,$$

since $0 < \pi_{Kiit} < 1$, $0 < \pi_{Kjjt} < 1$, and $\lambda_{Ajt} + \lambda_{Njt} = 1$, where v_t is the common real income across all populated locations.

Since $\mathbf{\Omega_t}(\mathbf{w_t})$ exhibits gross substitution in $\mathbf{w_t}$, this system of equations has at most one (normalized) solution. Gross substitution implies that $\mathbf{\Omega_t}(\mathbf{w_t}) = \mathbf{\Omega_t}(\mathbf{w_t'})$ cannot occur whenever $\mathbf{w_t}$ and $\mathbf{w_t'}$ are two wage vectors that are not colinear. Since $\Omega_{it}(\mathbf{w_t})$ is homogeneous of degree zero in $\{\mathbf{w_t}\}$, we can assume $\mathbf{w_t'} \geq \mathbf{w_t}$ and $w_{it} = w_{it}'$ for some i. Now consider altering the wage vector $\mathbf{w_t'}$ to obtain the wage vector $\mathbf{w_t}$ in $I-1$ steps, lowering (or keeping unaltered) the wage of all the other $I-1$ locations $s \neq i$ one at a time. By gross substitution, $\Omega_{it}(\mathbf{w_t})$ cannot decrease in any step, and because $\mathbf{w_t} \neq \mathbf{w_t'}$, it will actually increase in at least one step. Hence $\Omega_{it}(\mathbf{w_t}) > \Omega_{it}(\mathbf{w_t'})$ and we have a contradiction.

We next establish that there exists a wage vector $\mathbf{w_t^*} \in \Re_+^S$ such that $\mathbf{\Omega_t}(\mathbf{w_t^*}) = 0$. By homogeneity of degree zero, we can restrict our search for an equilibrium wage vector to the unit simplex $\Delta = \left\{\mathbf{w_t} \in \Re_+^S : \sum_{i=1}^{I} w_{it} = 1\right\}$. Define on Δ the function $\Omega_{it}^+(\cdot)$ by $\Omega_{it}^+(\mathbf{w_t}) = \max\{\Omega_{it}(\mathbf{w_t}), 0\}$. Note that $\Omega_{it}^+(\cdot)$ is continuous. Denote $\varpi_t(\mathbf{w_t}) = \sum_{i=1}^{I}\left[w_{it} + \Omega_{it}^+(\mathbf{w_t})\right]$. We have $\varpi_t(\mathbf{w_t}) \geq 1$ for all $\mathbf{w_t}$. Define a continuous function $\boldsymbol{\varsigma}_\mathbf{t}(\cdot)$ from the closed convex set Δ into itself by:

$$\boldsymbol{\varsigma}_\mathbf{t}(\mathbf{w_t}) = \left[1/\varpi_t(\mathbf{w_t})\right]\left[\mathbf{w_t} + \mathbf{\Omega_t^+}(\mathbf{w_t})\right].$$

Note that this fixed-point function tends to increase the wages of locations with $\Omega_{it}(\mathbf{w_t}) > 0$. By Brouwer's Fixed-point Theorem, there exists $\mathbf{w_t^*} \in \Delta$ such that $\mathbf{w_t^*} = \boldsymbol{\varsigma}_\mathbf{t}(\mathbf{w_t^*})$. Since

$\sum_{i=1}^{I} \Omega_{it}(\mathbf{w_t}) = 0$, it cannot be the case that $\Omega_{it}(\mathbf{w_t}) > 0$ for all $i = 1, \ldots, I$ or $\Omega_{it}(\mathbf{w_t}) < 0$ for all $i = 1, \ldots, I$. Additionally, if $\Omega_{it}(\mathbf{w_t}) > 0$ for some i and $\Omega_{st}(\mathbf{w_t}) < 0$ for some $s \neq i$, $\mathbf{w_t} \neq \boldsymbol{\varsigma}_\mathbf{t}(\mathbf{w_t})$. It follows that at the fixed point for wages, $\mathbf{w_t^*} = \boldsymbol{\varsigma}_\mathbf{t}(\mathbf{w_t^*})$, $\Omega_{it}(\mathbf{w_t}) = 0$ for all i. ■

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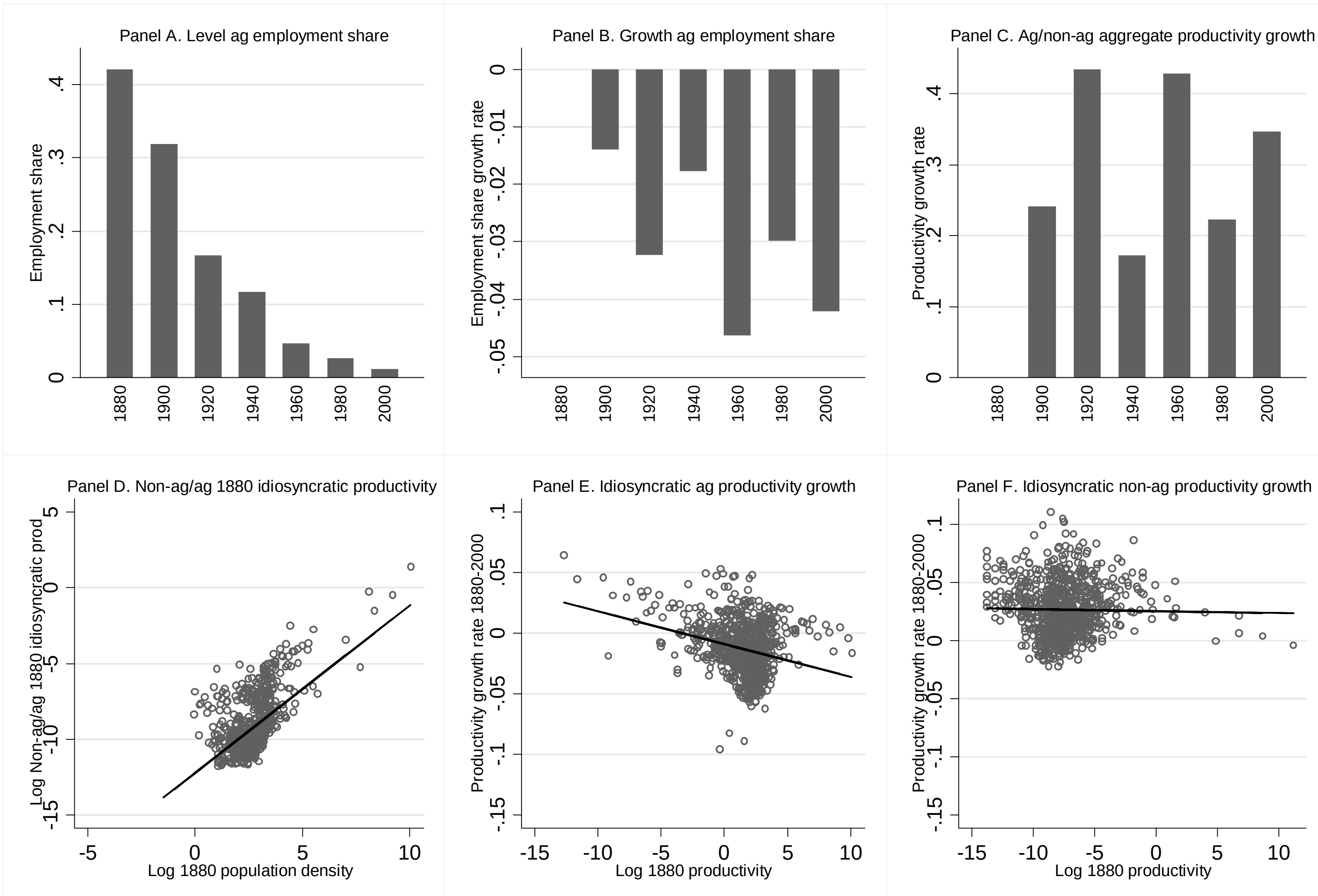
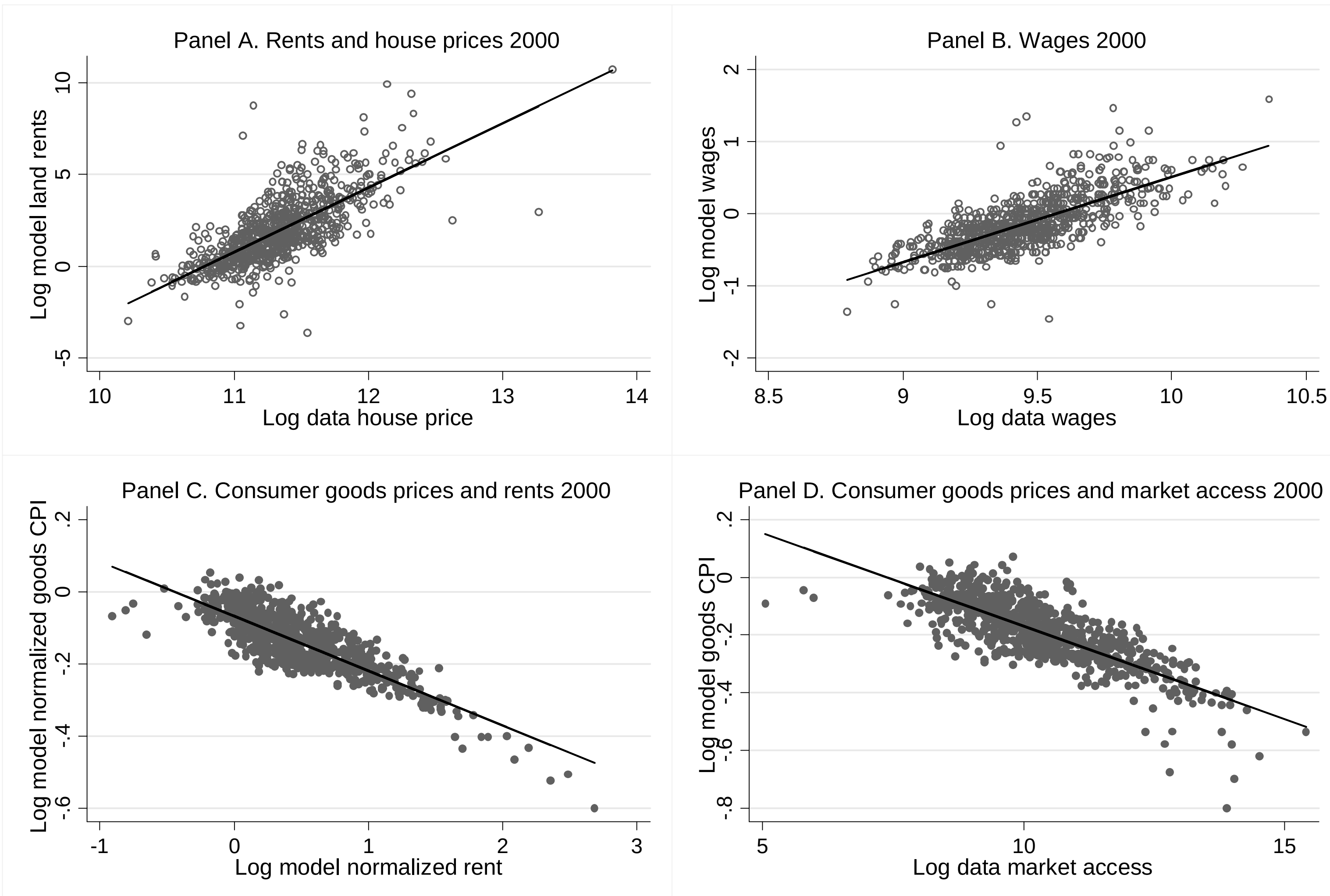
Figure 1: Employment shares and calibrated productivities



Note: See the text for further discussion of the construction of each panel of the figure.

Figure 2: Wages, rents and consumer goods prices



Note: See the text for further discussion of the construction of each panel of the figure.